MEM202 Engineering Mechanics – Statics
Final Examination **Solution**

Friday, September 02, 2005

1:00 PM – 3:00 PM

I. Solve all seven problems
II. Read the problems carefully.
III. Extra credit is 5 points.
IV. Equations you may need are given on the last page.

NAME: ______________________________

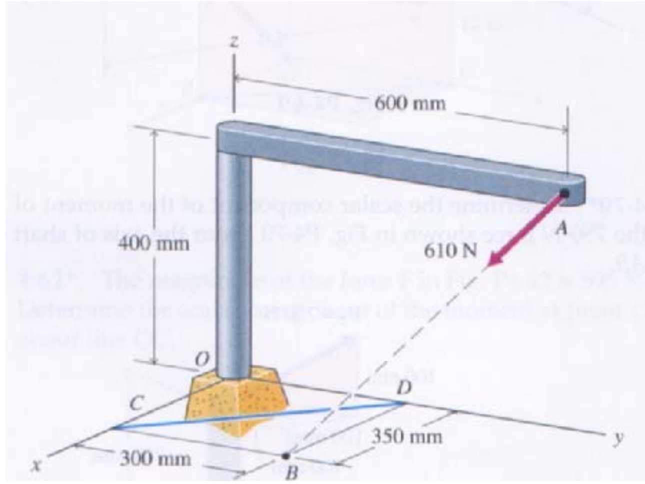
I.D.: _________________________________

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______

Extra credit:______

Total______

1. (15 Points) Determine the moment of the 610 N force shown about line CD.
Express the result in Cartesian vector form.



SOLUTION

$$\vec{F} = 610\left[\frac{350\ \hat{\imath} - 300\ \hat{\jmath} - 400\ \hat{k}}{\sqrt{(350)^2 + (-300)^2 + (-400)^2}}\right]$$

$$= 349.8\ \hat{\imath} - 299.8\ \hat{\jmath} - 399.8\ \hat{k}\ \ \text{N}$$

$$\vec{r}_{B/C} = 0.300\ \hat{\jmath}\ \ \text{m}$$

$$\vec{M}_C = \vec{r}_{B/C} \times \vec{F} = (0.300\ \hat{\jmath}) \times (349.8\ \hat{\imath} - 299.8\ \hat{\jmath} - 399.8\ \hat{k})$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 0.300 & 0 \\ 349.8 & -299.8 & -399.8 \end{vmatrix} = -119.94\ \hat{\imath} - 104.94\ \hat{k}\ \ \text{N·m}$$

$$\hat{e}_{CD} = \frac{-350\ \hat{\imath} + 300\ \hat{\jmath}}{\sqrt{(-350)^2 + (300)^2}} = -0.7593\ \hat{\imath} + 0.6508\ \hat{\jmath}$$

$$M_{CD} = \vec{M}_C \cdot \hat{e}_{CD} = (-119.94\ \hat{\imath} - 104.94\ \hat{k}) \cdot (-0.7593\ \hat{\imath} + 0.6508\ \hat{\jmath})$$

$$= 91.07\ \text{N·m} \cong 91.1\ \text{N·m}$$

$$\vec{M}_{CD} = M_{CD}\hat{e}_{CD} = 91.07(-0.7593\ \hat{\imath} + 0.6508\ \hat{\jmath})$$

$$= -69.1\ \hat{\imath} + 59.3\ \hat{\jmath}\ \ \text{N·m}$$ Ans.

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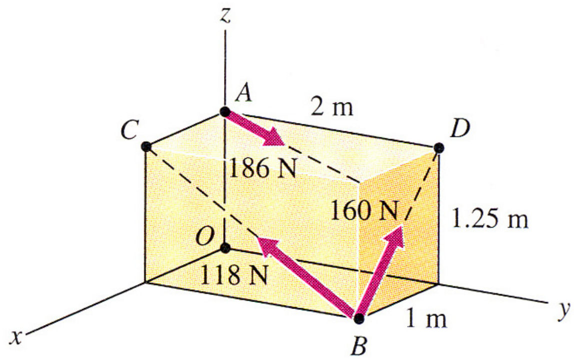
2. (10 Points) Reduce the forces shown to a wrench and locate the intersection of the wrench with the x-y plane.

Hint: A wrench is formed by $\vec{R}$ and $\vec{C}_{//}$ where $\vec{C}_{//}$ is the vector component of $\vec{C}$ parallel to $\vec{R}$. The location of a wrench, in terms of a vector $\vec{r} = x_R\vec{i} + y_R\vec{j}$ on the *x-y* plane, can be determined by using $\vec{r} \times \vec{R} = \vec{C}_{\perp}$ where $\vec{C}_{\perp} = \vec{C} - \vec{C}_{//}$.



$$F_{186} = 83.2\vec{i} + 166.4\vec{j} \text{ N}$$

$$F_{118} = -100.1\vec{j} + 62.5\vec{k} \text{ N}$$

$$F_{160} = -99.95\vec{i} + 124.9\vec{k} \text{ N}$$

SOLUTION

$$\mathbf{F}_A = 186\left[\frac{1\ \hat{\imath} + 2\ \hat{\jmath}}{\sqrt{5}}\right] = 83.18\ \hat{\imath} + 166.36\ \hat{\jmath}\ \text{N}$$

$$\mathbf{F}_B = 118\left[\frac{-2\ \hat{\jmath} + 1.25\ \hat{k}}{\sqrt{5.5625}}\right] + 160\left[\frac{-1\ \hat{\imath} + 1.25\ \hat{k}}{\sqrt{2.5625}}\right]$$

$$= -99.95\ \hat{\imath} - 100.06\ \hat{\jmath} + 187.48\ \hat{k}\ \text{N}$$

$$\bar{\mathbf{R}} = \Sigma\mathbf{F} = \mathbf{F}_A + \mathbf{F}_B = -16.77\ \hat{\imath} + 66.30\ \hat{\jmath} + 187.48\ \hat{k}\ \text{N} \qquad \text{Ans.}$$

$$R = \sqrt{(-16.77)^2 + (66.30)^2 + (187.48)^2} = 199.56\ \text{N}$$

$$\bar{\mathbf{C}} = \Sigma\bar{\mathbf{M}}_O = (\bar{\mathbf{r}}_{A/O} \times \mathbf{F}_A) + (\bar{\mathbf{r}}_{B/O} \times \mathbf{F}_B)$$

$$= [(1.25\ \hat{k}) \times (83.18\ \hat{\imath} + 166.36\ \hat{\jmath})]$$

$$+ [(1\ \hat{\imath} + 2\ \hat{\jmath}) \times (-99.95\ \hat{\imath} - 100.06\ \hat{\jmath} + 187.48\ \hat{k})]$$

$$= 167.01\ \hat{\imath} - 83.51\ \hat{\jmath} + 99.84\ \hat{k}\ \text{N}\cdot\text{m}$$

$$\hat{\mathbf{e}}_R = \frac{-16.77}{199.56}\ \hat{\imath} + \frac{66.30}{199.56}\ \hat{\jmath} + \frac{187.48}{199.56}\ \hat{k} = -0.0840\ \hat{\imath} + 0.3322\ \hat{\jmath} + 0.9395\ \hat{k}$$

$$C_{\parallel} = \bar{\mathbf{C}}\cdot\hat{\mathbf{e}}_R = (167.01\ \hat{\imath} - 83.51\ \hat{\jmath} + 99.84\ \hat{k})\cdot(-0.0840\ \hat{\imath} + 0.3322\ \hat{\jmath} + 0.9395\ \hat{k})$$

$$= 52.03\ \text{N}\cdot\text{m}$$

$$\bar{\mathbf{C}}_{\parallel} = C_{\parallel}\hat{\mathbf{e}}_R = 52.03(-0.0840\ \hat{\imath} + 0.3322\ \hat{\jmath} + 0.9395\ \hat{k})$$

$$= -4.371\ \hat{\imath} + 17.28\ \hat{\jmath} + 48.88\ \hat{k}\ \text{N}\cdot\text{m} \qquad \text{Ans.}$$

$$\bar{\mathbf{C}}_{\perp} = \bar{\mathbf{C}} - \bar{\mathbf{C}}_{\parallel} = 171.38\ \hat{\imath} - 100.79\ \hat{\jmath} + 50.96\ \hat{k}\ \text{N}\cdot\text{m}$$

$$\bar{\mathbf{C}}_{\perp} = \bar{\mathbf{r}} \times \bar{\mathbf{R}} = (x_R\ \hat{\imath} + y_R\ \hat{\jmath}) \times (-16.77\ \hat{\imath} + 66.30\ \hat{\jmath} + 187.48\ \hat{k})$$

$$= 187.48\ y_R\ \hat{\imath} - 187.48\ x_R\ \hat{\jmath} + (66.3\ x_R + 16.77\ y_R)\ \hat{k}$$

From $\hat{\jmath}$: $\quad -187.48x_R = -100.79 \qquad x_R = 0.5376\ \text{m} \cong 538\ \text{mm}$ Ans.

From $\hat{\imath}$: $\quad 187.48y_R = 171.38 \qquad y_R = 0.9141\ \text{m} \cong 914\ \text{mm}$ Ans.

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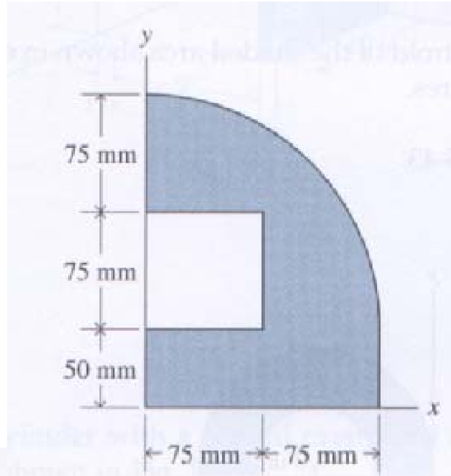
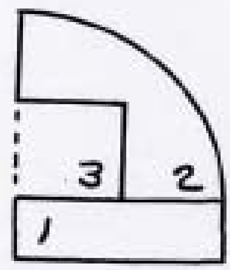
3. (15 Points) Locate the centroid (x_c, y_c) of the composite area shown.
Note: do not move the reference system.



SOLUTION

The shaded area can be divided into a rectangle, and a quarter circle, with a square removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$A_2 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(150)^2 = 17{,}671 \text{ mm}^2$$

$$x_{c2} = \frac{4r}{3\pi} = \frac{4(150)}{3\pi} = 63.66 \text{ mm}$$

$$y_{c2} = 50 + \frac{4r}{3\pi} = 50 + \frac{4(150)}{3\pi} = 113.66 \text{ mm}$$

Part	A_i (mm^2)	x_{ci} (mm)	M_y (mm^3)	y_{ci} (mm)	M_x (mm^3)
1	7500	75	562,500	25	187,500
2	17,671	63.66	1,124,936	113.66	2,008,486
3	-5625	37.5	-210,938	87.5	-492,188
Σ	19,546		1,476,498		1,703,798

$A x_c = \Sigma A_i x_{ci} = M_y$ $\qquad x_c = \frac{M_y}{A} = \frac{1{,}476{,}498}{19{,}546} = 75.5 \text{ mm}$ Ans.

$A y_c = \Sigma A_i y_{ci} = M_x$ $\qquad y_c = \frac{M_x}{A} = \frac{1{,}703{,}798}{19{,}546} = 87.2 \text{ mm}$ Ans.

$X_C = 75.7mm, Y_C = 87.2mm$

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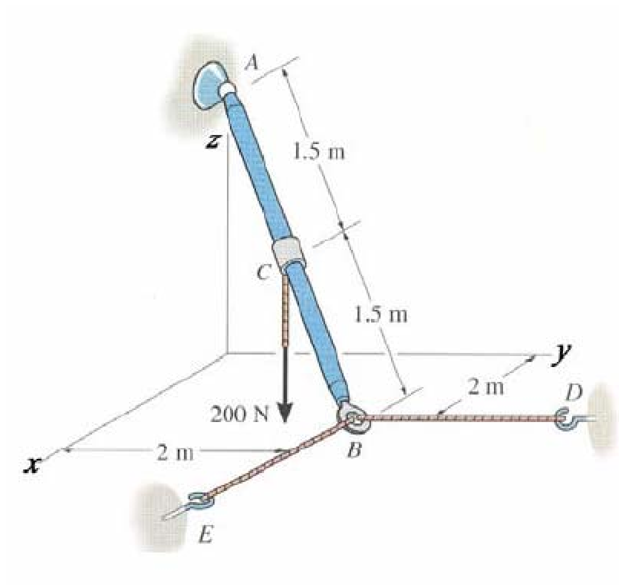
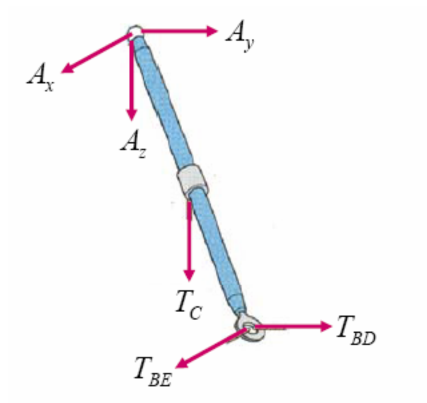
4. (15 Points) Rod ABC is subjected to a vertical force of 200 N at C and is supported at A by a ball-and-socket joint (i.e., it can transmit force but no moment) and cables BD (parallel to y-axis) and BE (parallel to x-axis) at B. Determine the reaction force at A (Ax, Ay and Az) and forces in the two cables.
Note: you must draw free-body diagram for rod ABC.



<u>Solution</u>
The free body diagram of rod ABC is shown on the right. Balance of forces yields



(i) $$\sum \overline{F} = \overline{A} + \overline{T}_c + \overline{T}_{BE} + \overline{T}_{BD} = 0$$

And balance of moments about A yield

(ii) $$\sum \overline{M}_A = \overline{r}_{C/A} \times \overline{T}_c + \overline{r}_{B/A} \times \left(\overline{T}_{BE} + \overline{T}_{BD}\right) = 0$$

Where

(iii) $\overline{A} = A_X\,\overline{i} + A_Y\,\overline{j} + A_Z\,\overline{k}$

(iv) $\overline{T}_C = -200\overline{k}N$

(v) $\overline{T}_{BE} = T_{BE}\,\overline{i}N$

(vi) $\overline{T}_{BD} = T_{BD}\,\overline{j}N$

(vii) $$\overline{r}_{C/A} = 1.5\left(\frac{2.0\overline{i} + 2.0\overline{j} - 1.0\overline{k}}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}\right) = 1.0\overline{i} + 1.0\overline{j} - 0.5\overline{k}$$

(viii) $$\overline{r}_{B/A} = 3.0\left(\frac{2.0\overline{i} + 2.0\overline{j} - 1.0\overline{k}}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}\right) = 2.0\overline{i} + 2.0\overline{j} - 1.0\overline{k}$$

Substituting (iii) – (vi) in (i) gives

(ix) $A_X = -T_{BE}$, $A_Y = -T_{BD}$, $A_Z = 200N$

Substituting (vii), (viii) and (ix) in (ii) gives

$$\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1.0 & 1.0 & -0.5 \\ 0 & 0 & -200 \end{vmatrix} + \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 2.0 & 2.0 & -1.0 \\ T_{BE} & T_{BD} & 0 \end{vmatrix} = \left(-200 - T_{BD}\right)\overline{i} + \left(200 + T_{BE}\right) + \left(2T_{BD} - 2T_{BE}\right)$$

Thus,

$$A_X = 200N, A_Y = 200N, A_Z = 200N, T_{BE} = -200N, T_{BD} = -200N$$

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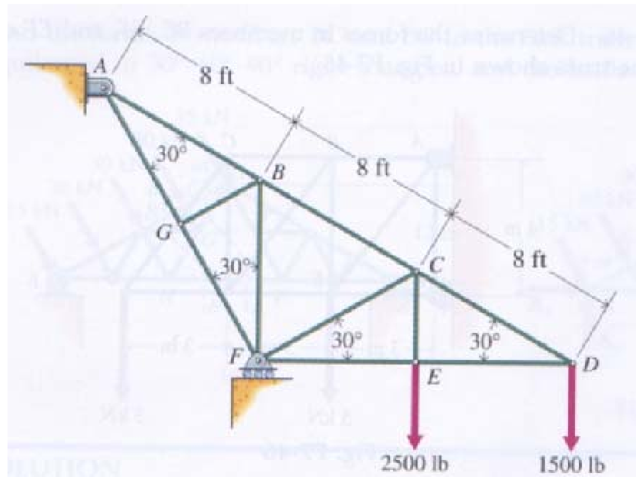
5.



a) (5 Points) Identify all zero force members in the truss shown.

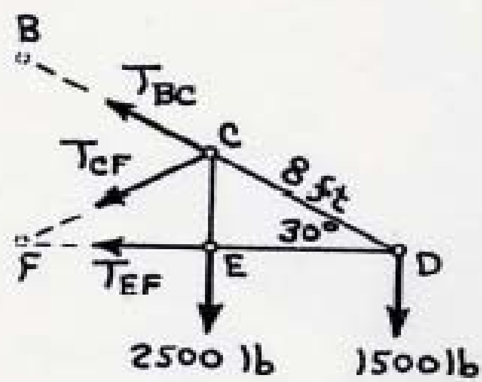
GB, BF

b) (15 Points) Determine the forces in members BC, CF and EF of the truss shown.
Hint: use METHOD OF SECTIONS

SOLUTION

For this simple truss, the member forces can be determined without solving for the support reactions.

From a free-body diagram of the part of the truss to the right of member BF:



$$+\circlearrowleft \Sigma M_C = -T_{EF}(8 \sin 30^\circ) - 1500(8 \cos 30^\circ) = 0$$

$$T_{EF} = -2598 \text{ lb} \cong 2600 \text{ lb (C)}$$ Ans.

$$+\circlearrowleft \Sigma M_F = T_{BC} \cos 30^\circ (8 \sin 30^\circ) + T_{BC} \sin 30^\circ (8 \cos 30^\circ)$$
$$- 2500(8 \cos 30^\circ) - 1500(16 \cos 30^\circ) = 0$$

$$T_{BC} = 5500 \text{ lb} = 5500 \text{ lb (T)}$$ Ans.

$$+\circlearrowleft \Sigma M_D = T_{CF} \cos 30^\circ (8 \sin 30^\circ) + T_{CF} \sin 30^\circ (8 \cos 30^\circ)$$
$$+ 2.500(8 \cos 30^\circ) = 0$$

$$T_{CF} = -2500 \text{ lb} = 2500 \text{ lb (C)}$$ Ans.

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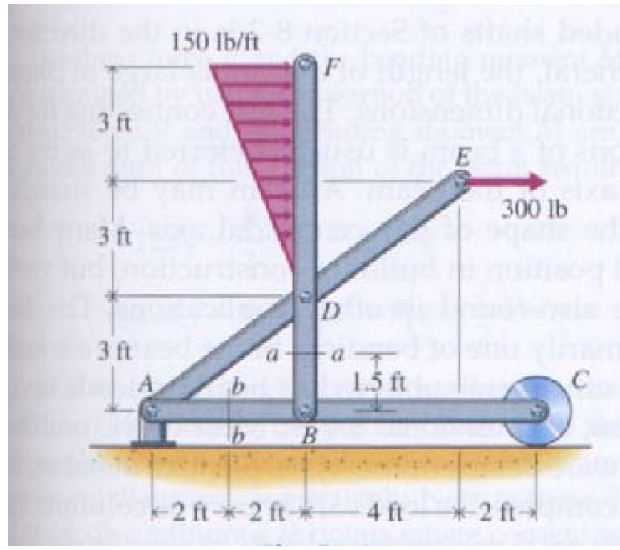
6. (15 points) A three-bar frame is loaded and supported as shown. Determine the internal resisting forces and moment transmitted by:

a. Section *aa* in bar BDF
b. Section *bb* in bar ABC

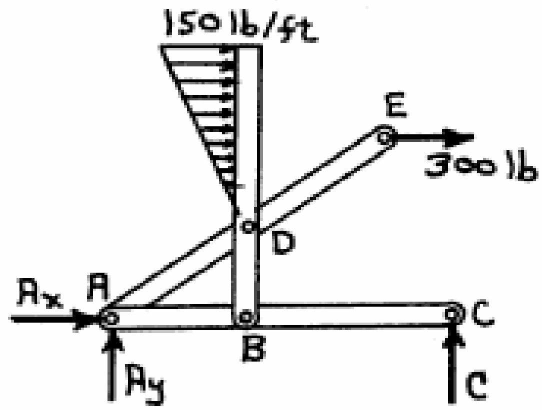


SOLUTION

From a free-body diagram for the complete frame:

$$+\circlearrowleft \Sigma M_A = C(10) - 300(6) - \frac{1}{2}(150)(6)(7) = 0$$

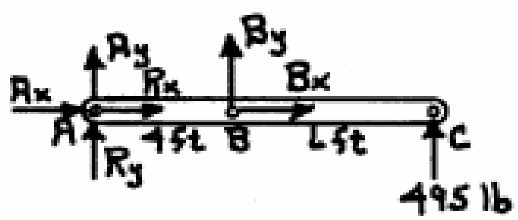
$$C = 495 \text{ lb} = 495 \text{ lb} \uparrow$$



From a free-body diagram for bar ABC:

$$+\circlearrowleft \Sigma M_A = B_y(4) + 495(10) = 0$$

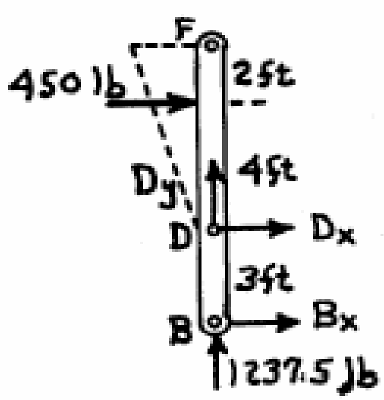
$$B_y = -1237.5 \text{ lb} = 1237.5 \text{ lb} \downarrow$$



From a free-body diagram for bar BDF:

$$+\circlearrowleft \Sigma M_D = B_x(3) - \frac{1}{2}(150)(6)(4) = 0$$

$$B_x = 600 \text{ lb} = 600 \text{ lb} \longrightarrow$$



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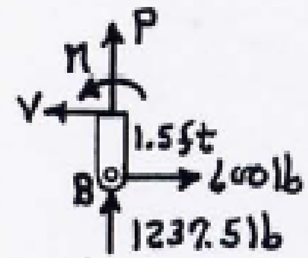
(a) From a free-body diagram of bar BDF below section aa:



$$+\uparrow \Sigma F_n = P + 1237.5 = 0$$

$$P = -1237.5 \text{ lb} \cong 1238 \text{ lb} \downarrow$$ Ans.

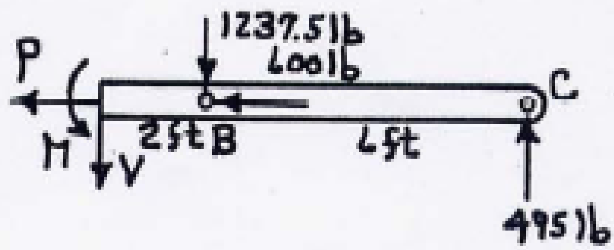
$$+\leftarrow \Sigma F_t = V - 600 = 0$$

$$V = 600 \text{ lb} = 600 \text{ lb} \leftarrow$$ Ans.

$$+\circlearrowleft \Sigma M_z = M + 600(1.5) = 0$$

$$M = -900 \text{ ft}\cdot\text{lb} = 900 \text{ ft}\cdot\text{lb} \circlearrowright$$ Ans.

(b) From a free-body diagram of bar ABC to the right of section bb:



$$+\leftarrow \Sigma F_n = P + 600 = 0$$

$$P = -600 \text{ lb} = 600 \text{ lb} \rightarrow$$ Ans.

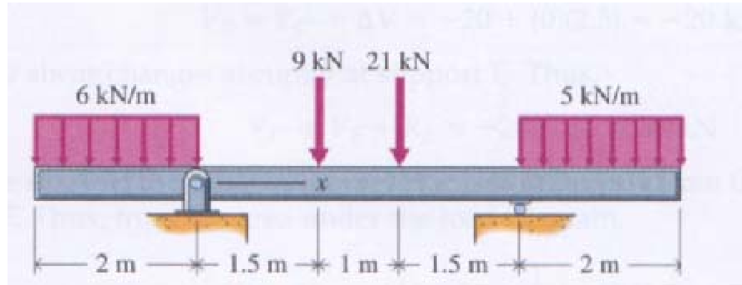
$$+\downarrow \Sigma F_t = V + 1237.5 - 495 = 0$$

$$V = -742.5 \text{ lb} \cong 743 \text{ lb} \uparrow$$ Ans.

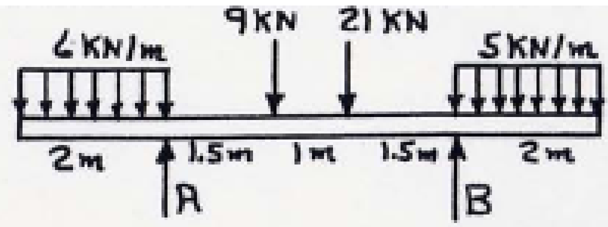
$$\circlearrowleft + \Sigma M_z = M - 1237.5(2) + 495(8) = 0$$

$$M = -1485 \text{ ft}\cdot\text{lb} = 1485 \text{ ft}\cdot\text{lb} \circlearrowright$$ Ans.

7. (15 Points) Draw complete shear and bending moment diagrams for the beam shown. Write down shear force and bending moment equations for all segments of the beam, and mark clearly on the diagram the locations and magnitudes of the maximum and minimum moments. Any method is acceptable, but you must show your solution process in detail.



SOLUTION



From a free-body diagram for the complete beam:

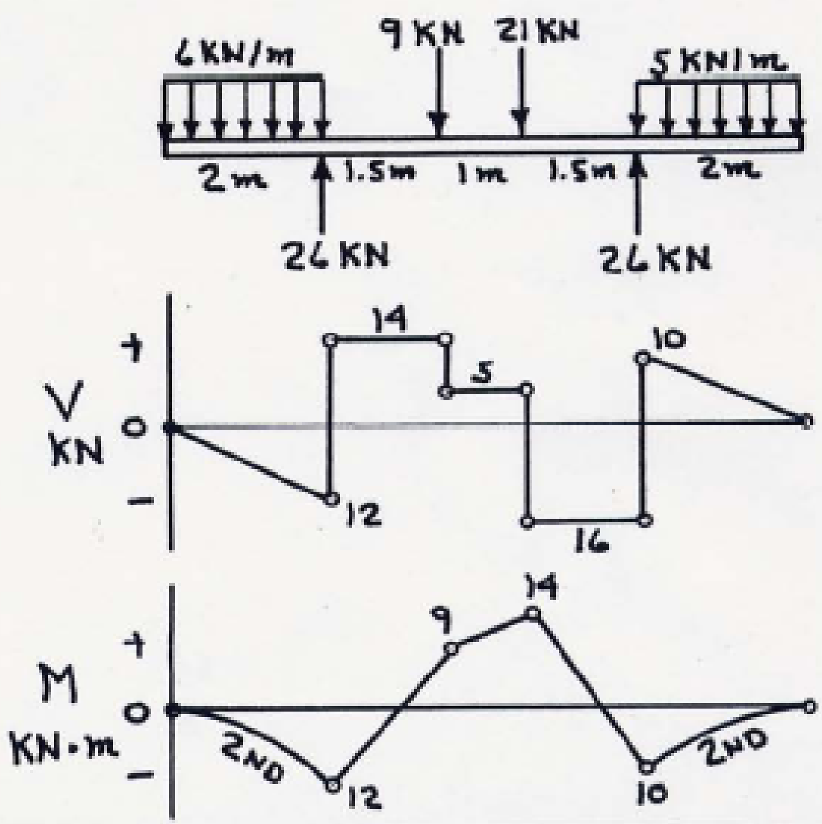
$$+\circlearrowleft \Sigma M_A = 2(6)(1) - 9(1.5) - 21(2.5) + B(4) - 5(2)(5) = 0$$

$$B = 26 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 2(6)(5) - A(4) + 9(2.5) + 21(1.5) - 5(2)(1) = 0$$

$$A = 26 \text{ kN}$$

Load, shear, and moment diagrams for the beam are shown below:



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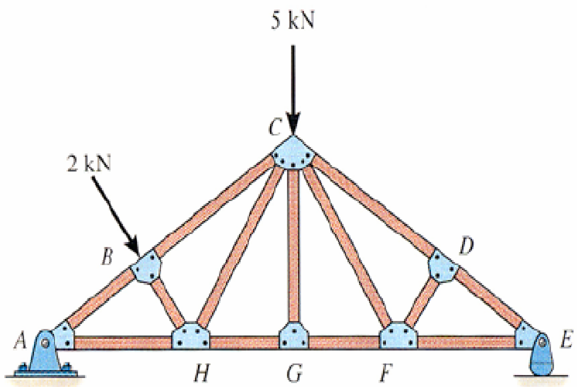
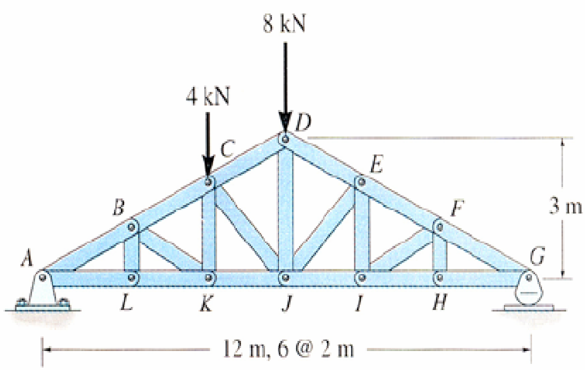
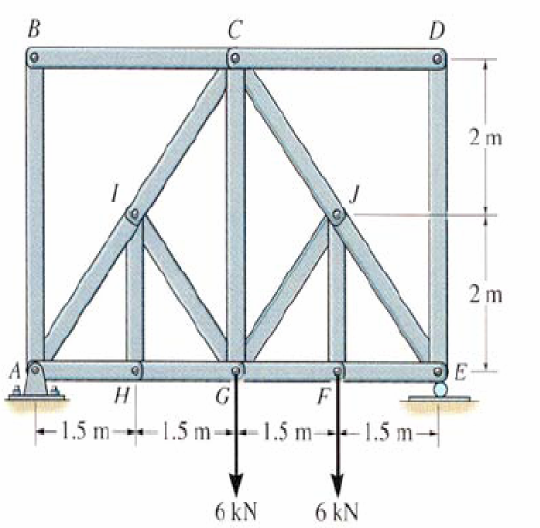
Extra Credit Extra Credit Extra Credit

(5 Points) Identify ALL zero forces members in each truss shown.

Truss	Zero-force members
	Joint $D \Rightarrow DF = 0$ Joint $F \Rightarrow CF = 0$ Joint $G \Rightarrow CG = 0$
	Joint $H \Rightarrow FH = 0$ Joint $F \Rightarrow FI = 0$ Joint $I \Rightarrow EI = 0$ Joint $E \Rightarrow EJ = 0$ Joint $L \Rightarrow BLF = 0$ Joint $B \Rightarrow BK = 0$ Joint $K \Rightarrow CK = 0$
	Joint $B \Rightarrow AB = BC = 0$ Joint $D \Rightarrow CD = DE = 0$ Joint $H \Rightarrow HI = 0$ Joint $I \Rightarrow GI = 0$

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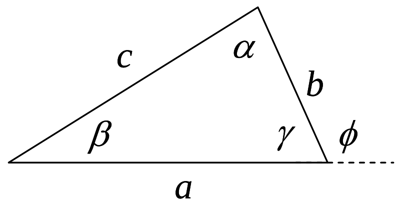
Equations you may need:

a. Law of sine and law of cosine:



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$\text{or } c^2 = a^2 + b^2 + 2ab\cos\phi$$

b. Products of vectors:

Given two vectors $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$ and $\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$:

- The dot product of the two vectors is
$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = |A||B|\cos\left(\text{angle between } \vec{A} \text{ and } \vec{B}\right)$$
- The cross product of the two vectors is
$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \left(A_yB_z - A_zB_y\right)\vec{i} + \left(A_zB_x - A_xB_z\right)\vec{j} + \left(A_xB_y - A_yB_x\right)\vec{k}$$

NOTE:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{matrix} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ A_x & A_y & A_z & A_x & A_y \\ B_x & B_y & B_z & B_x & B_y \end{matrix}$$
$$- \quad - \quad - \quad + \quad + \quad +$$

c. A vector can be expressed either in terms of its magnitude and direction, i.e., $\vec{A} = |A|\vec{e}_A$, or in terms of its Cartesian components, i.e., $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$, where
$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad \vec{e}_A = \frac{A_x}{A}\vec{i} + \frac{A_y}{A}\vec{j} + \frac{A_z}{A}\vec{k}$$

d. Centroid of a quarter circle:
$$x_c = \frac{4r}{3\pi}$$
$$y_c = \frac{4r}{3\pi}$$

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